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WAEP Semester One Examination, 2020

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures

In words ______________________

Your name ______________________

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(5 marks)**

The function $f(z)$ is of degree 4 and has factors $z - 4 - i$ and $z + 3i$.

(a) Determine $f(z)$ in the form $z^4 + az^3 + bz^2 + cz + d$, where $\{a, b, c, d\} \in \mathbb{R}$. (3 marks)

(b) Explain whether your answer to part (a) would change if the coefficients of the polynomial $f(z)$ were not restricted to real numbers. (2 marks)

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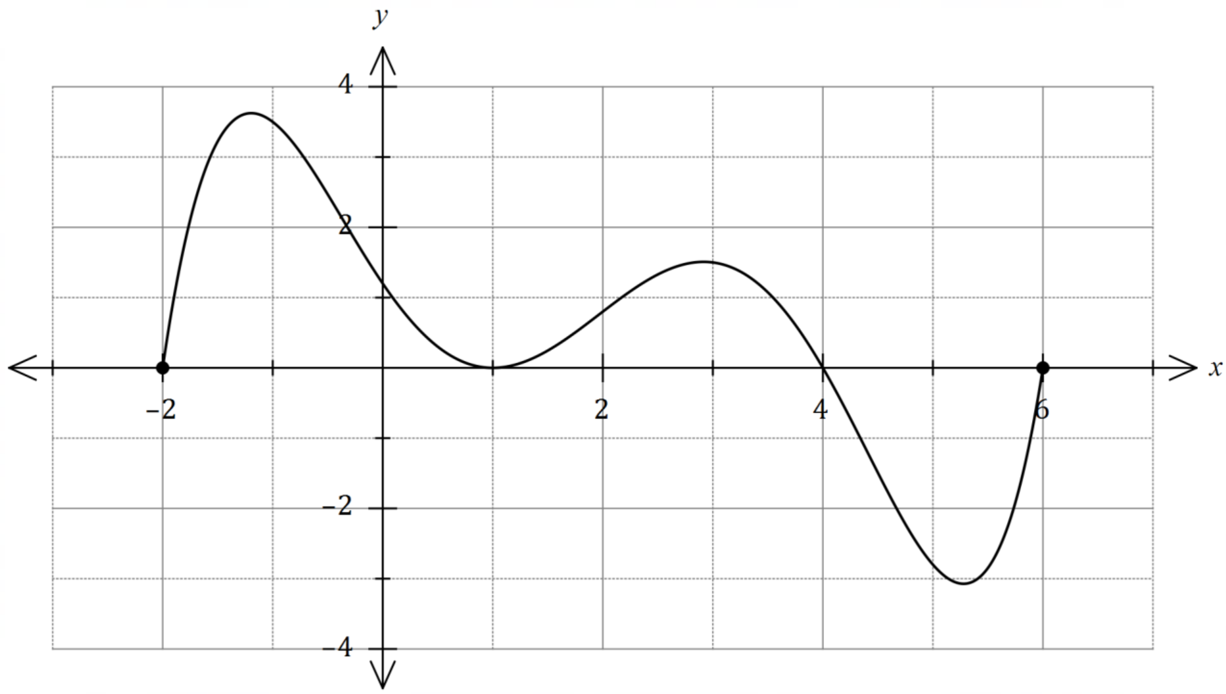
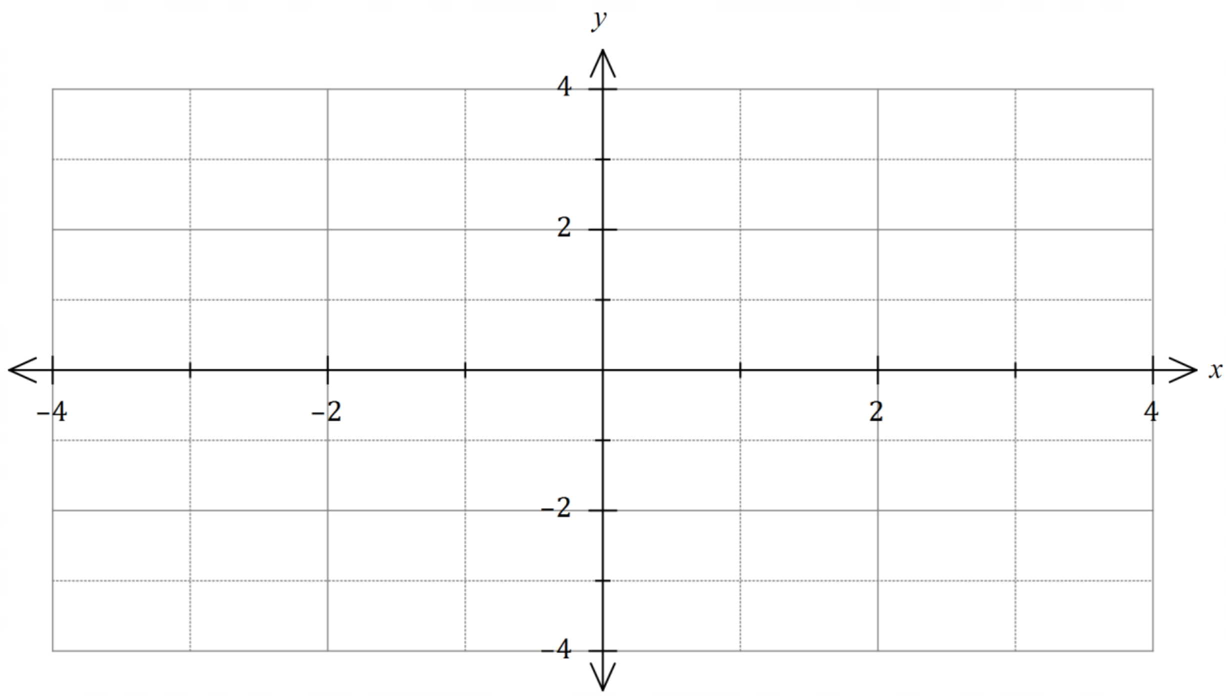
Question 10 **(8 marks)**

The graph of $y = f(x)$ is shown below over the domain $-2 \leq x \leq 6$.



(a) Sketch the graph of $y = f(|x|)$ over the domain $-3 \leq x \leq 3$ on the axes below. (2 marks)



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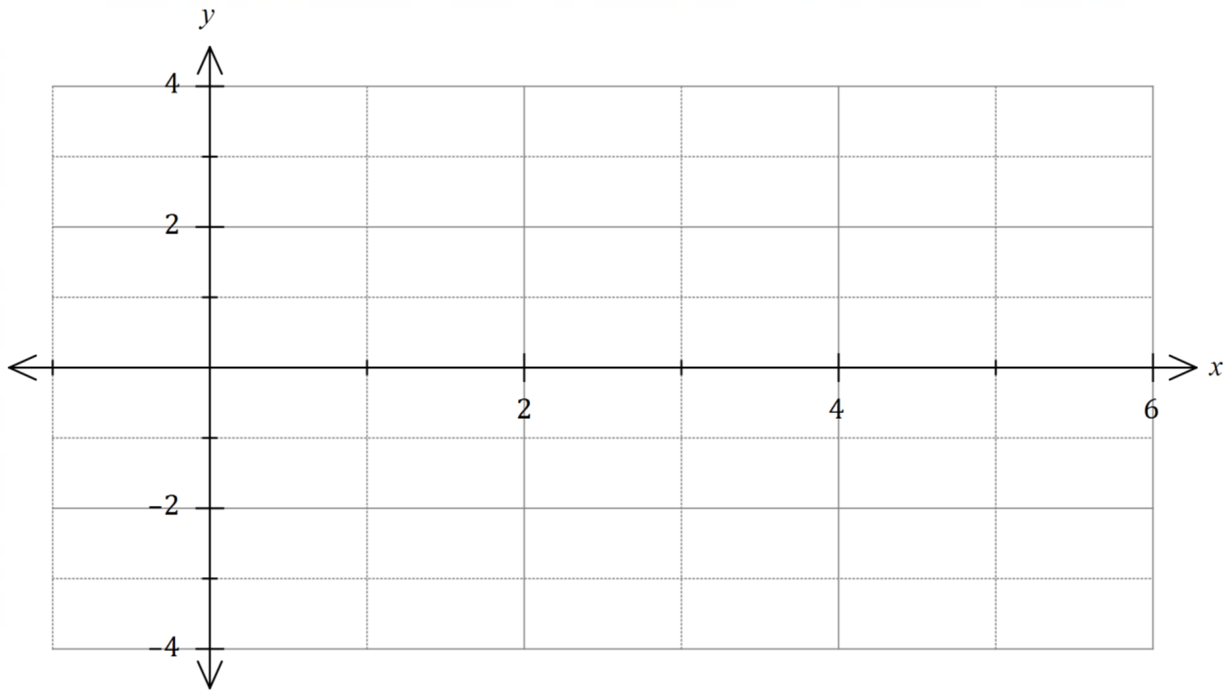
(b) Sketch the graph of $y = \dfrac{1}{f(x)}$ on the axes below over the domain $0 \leq x \leq 5$. (4 marks)



(c) List the equations of all asymptotes of the graph of $y = \dfrac{1}{f(|x|)}$ when drawn over the domain $-6 \leq x \leq 6$. (2 marks)

Question 11 **(8 marks)**

Drone A and drone B move with constant velocities and relative to the origin O have initial positions $(-4, 22, 2)$ and $(5, 15, 3)$ respectively, where distances are in metres.

One second later, the position of A is $(-1, 20, 3)$ and the position of B is $(1, 14, 8)$.

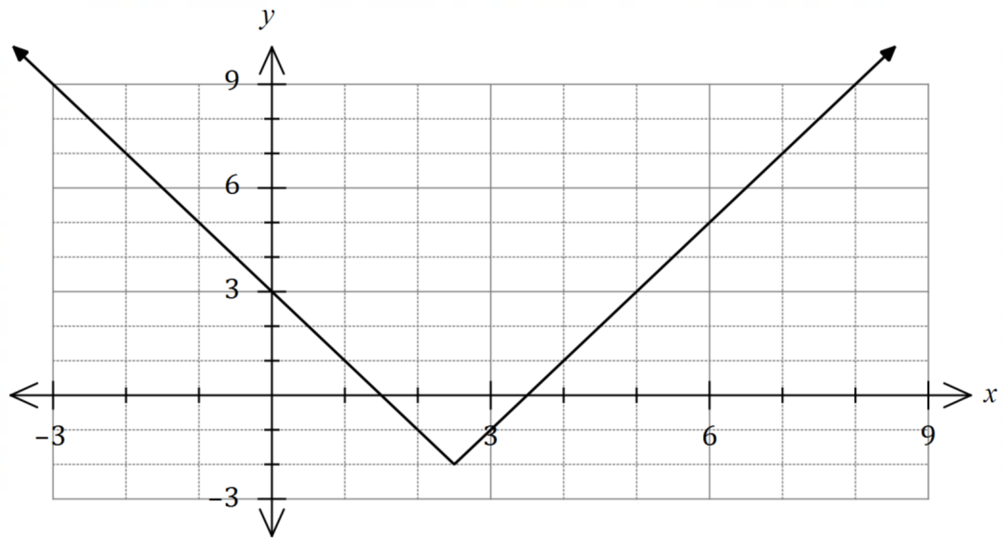
(a) Determine a position vector relative to the origin for each drone after t seconds. (3 marks)

(b) Determine an expression for the distance between the two drones at any time t, $t \geq 0$. (3 marks)

(c) Determine the minimum distance between the drones. (2 marks)

Question 12 **(7 marks)**

The graph of $f(x) = |ax + b| + c$ is shown below.



(a) Determine all possible values of the constants a, b and c. (3 marks)

(b) Using the graph, or otherwise, solve

(i) $f(x) = 5$. (1 mark)

(ii) $f(x) = x$. (1 mark)

(iii) $f(x) + 2x = 3$. (2 marks)

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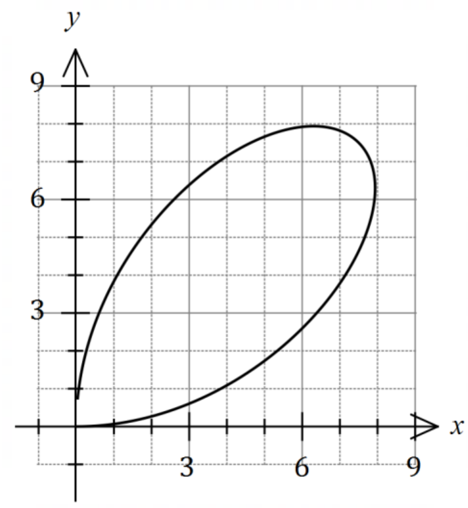
Question 13 **(9 marks)**

The path of a particle with position vector $\mathbf{r}(t) = \dfrac{15t}{1+t^3}\mathbf{i} + \dfrac{15t^2}{1+t^3}\mathbf{j}$ metres is shown below, where t is the time in seconds and $t \geq 0$.



(a) Determine the initial velocity of the particle. (2 marks)

(b) Determine the velocity of the particle at the instant, $t > 0$, when it is moving parallel to the x-axis. (2 marks)

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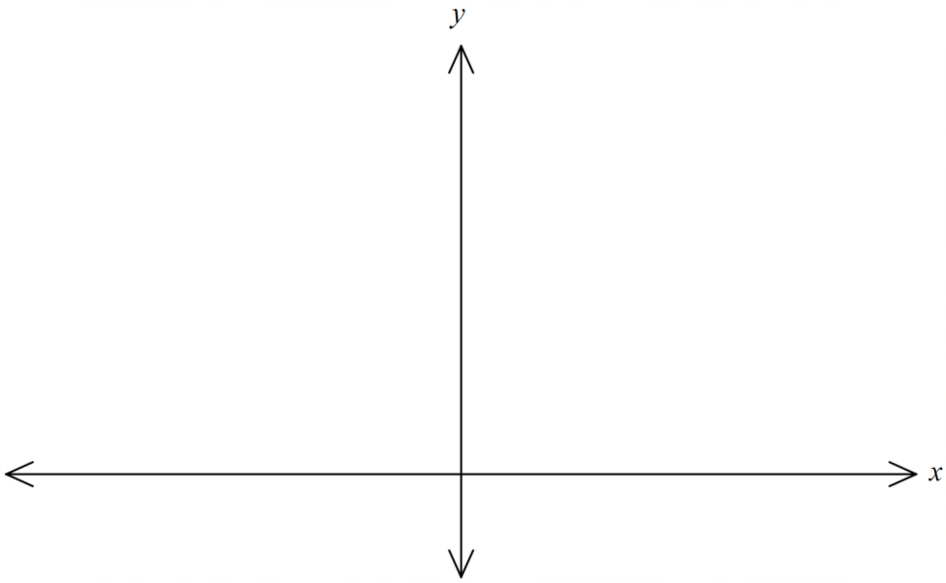
(c) Explain whether the particle will return to its initial position. (2 marks)

(d) Observing that $y - xt = 0$, show that the Cartesian equation for the path of the particle can be expressed in the form $x^3 + y^3 = kxy$ and state the value of the constant k.
(3 marks)

Question 14 **(7 marks)**

Let $f(x) = \left|\dfrac{x+2}{x-1}\right|$.

(a) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)

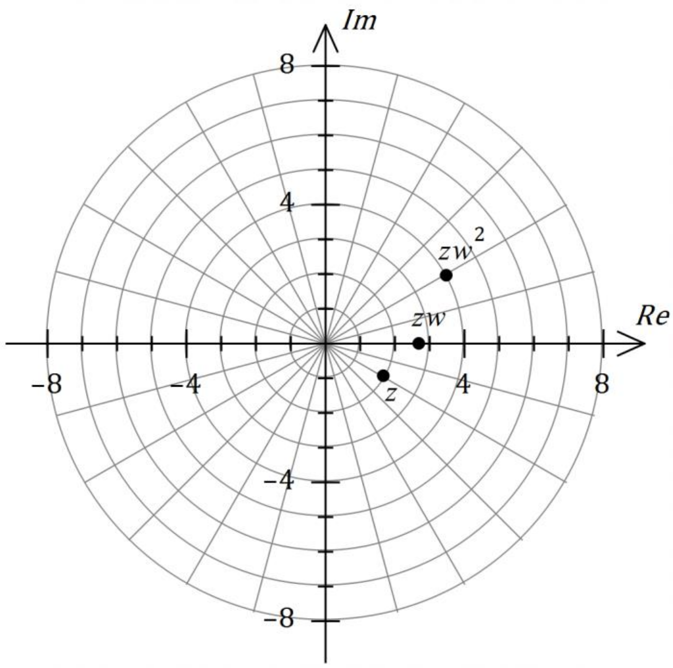


(b) State the range of $f(x)$. (1 mark)

(c) The domain of f is restricted to $-2 \leq x < b$ so that f^{-1} is a function. State the value of the constant b so that the domain of f is as large as possible and determine the domain and range for f^{-1}. (3 marks)

Question 15 **(8 marks)**

The complex numbers z, zw and zw^2 are represented on the Argand diagram below.



(a) Express z exactly in the form $a + bi$. (2 marks)

(b) Determine the modulus and argument of zw^5. (4 marks)

(c) Determine zw^{-2} and plot and label this point on the Argand diagram. (2 marks)

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Question 16 **(9 marks)**

(a) Let $f(x) = \dfrac{x^2 - 4x - 2}{x - 1}$.

(i) Briefly describe the feature of the rule for $f(x)$ that indicates the graph of $y = f(x)$ will have an oblique (slanted) asymptote. (1 mark)

(ii) Determine the equations of all asymptotes of the graph of $y = f(x)$. (3 marks)

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(b) Let $g(x) = \dfrac{(x-2)(x+3)}{x^2+1}$.

(i) State the equation of the horizontal asymptote of the graph of $y = g(x)$. (1 mark)

(ii) State the values of $g(6), g(7)$ and $g(8)$. (1 mark)

(iii) Use your previous two answers to explain why the graph of $y = g(x)$ must have a local maximum to the right of $x = 7$. (3 marks)

Question 17 **(8 marks)**

Plane Π has equation $\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

(a) Show how to deduce that the Cartesian equation of plane Π is $x - 5y - 3z = 1$. (3 marks)

The line through $A(1, 4, 5)$ and point B is perpendicular to Π, and the midpoint of AB lies in Π.

(b) Determine the coordinates of B. (5 marks)

Question 18 **(8 marks)**

(a) Determine, in the form $r\,\text{cis}\,\theta$, the solution of the equation $z^4 + 625i = 0$ that lies in the third quadrant of the complex plane ($-\pi < \theta < -\frac{\pi}{2}$). (4 marks)

(b) Writing $5 - 12i = (a + bi)^2,\ \{a, b\} \in \mathbb{R}$, or otherwise, use an algebraic method that does not involve CAS to determine the square roots of $5 - 12i$. (4 marks)

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Question 19 **(9 marks)**

The position vectors of particles A and B (in centimetres) at time t seconds, $t \geq 0$, are

$$\mathbf{r}_A = 7\mathbf{i} + 15\mathbf{j} + t(0.5\mathbf{i} - 2\mathbf{j}) \text{ and } \mathbf{r}_B = 5\mathbf{i} + 2\mathbf{j} + t\big((t-6)\mathbf{i} - \mathbf{j}\big).$$

(a) Show that A is moving with constant speed and determine this speed. (2 marks)

(b) Determine the Cartesian path of B. (3 marks)

(c) Determine the position vector of the point where the paths of the particles cross. (4 marks)

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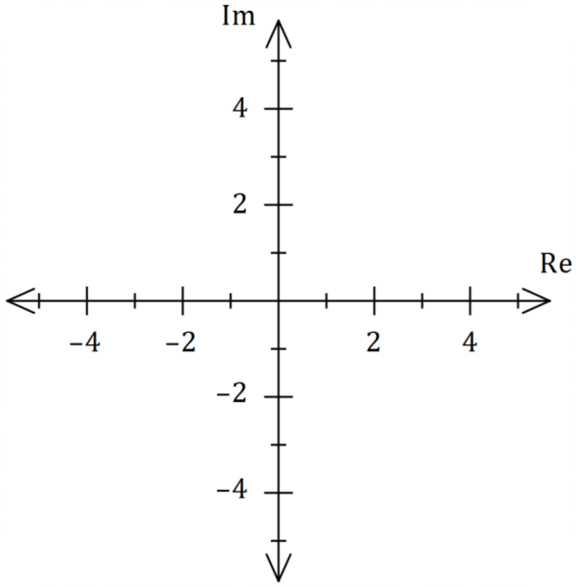
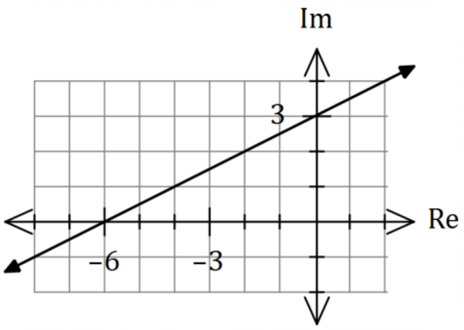
Question 20 **(8 marks)**

(a) Shade the region in the complex plane below that simultaneously satisfies $|z - 2i| \leq 3$ and $-\frac{\pi}{2} \leq \arg(z-1) \leq \frac{\pi}{2}$. (4 marks)



(b) The locus of $|z + 2i| = |z + a + bi|$ in the complex plane is the straight line shown below, $\{a, b\} \in \mathbb{R}$.



(i) State the value of constant a and the value of constant b. (2 marks)

(ii) Determine the minimum value of $|z|$ in exact form. (2 marks)

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Question 21 **(4 marks)**

Sphere S of radius 3 has its centre at the origin.

Line L has equation $\mathbf{r} = \begin{pmatrix} -k \\ k \\ -k \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, where k is a positive constant.

Prove that for L to be a tangent to S, then $k = \dfrac{3\sqrt{2}}{2}$.

End of questions

Supplementary page

Question number: _________